Dynamic stability of a five-link biped robot

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Abstract— **Keeping the dynamic stability during walking is one of the essential characteristics of regular bipedal walk, in the existence of unpowered DOF during SSP. To achieve the dynamic stability, there appears a decisive need to a robust controller to the robot movement. Here a new recurrent Neural Network is suggested as a controller for a five link biped robot, for tracking the desired angles trajectories for the legs of the robot.**

Keywords-biped robot; elman neural network; stability; control

I. INTRODUCTION

Walking is a fundamental feature of humanoid robots to achieve its goals, whatever these goals are, as the mobility of the robot is the main characteristic that categorizes it. For the biped robot there are three types of walkers (Marchese et al., 2001): static, dynamic and purely dynamic walkers. *Static walkers* are very slow walkers whose system's stability is completely described by the normal projection of the Centre of Gravity (COG), which depends on joints' position only, while *Dynamic walkers* have feet and actuated ankles. In this case the postural stability of dynamic walkers depends on joints' velocities and acceleration too. Dynamic walkers are potentially able to move in a static way, knowing that they have large feet and their motion is slow. Lastly, *purely dynamic walkers* are robots with no feet. In this case the contact area between the foot and the ground is reduced to a point, so that static walking is not possible. Hence normal human walking is a kind of dynamic bipedal locomotion, dynamic and purely dynamic walkers can simulate the human locomotion, which implies that, the five-link biped robot (as a purely dynamic walker) can emulate the human locomotion.

As mentioned by Vukobratovic and Juricic, 1969, the dynamic level collects the information on ambient and use these information for the purpose of control, through appropriate elements, to give the property of adaptability to the locomotion system. That means it is unavoidable to describe the gait of leg locomotion as a continuous process, based on the principles of analytical mechanics, using mathematical model. That's why the dynamic model of a biped robot is very important. On the other hand, practically every dynamic model has some degree of incorrectness and some errors in its parameters values, which cause errors in positioning and/or trajectory tracking which might cause system instability.

II. DYNAMIC MODEL

The main purpose of a modeling is to understand the system and to obtain a model that can be used to simulate and test the controllers. To model a biped robot there two main types of models: the kinematic model and the dynamic model. The kinematic model describes the motion of the biped robot without considering the exterior forces that cause the motion, while the dynamic model include all the exterior forces and is used to get the torques that act on each joint.

Many researchers contributed to the dynamic model of the biped robot, the main differences between these models are the number of links and degrees of freedom. Among these models, five-link biped robot has gained the attraction of many researchers (Furusho and Masubuchi, 1987, Tzafistas et al., 1996, Mu and Wu, 2004, Sadati and Hamed, 2007).

The biped under study consists of five links, one for the torso and two for each leg, those links are connected with four joints, two between the hip and each leg and two at the knees.

m_i: mass of link i.
l_i: length of link i.
r_i: is the distance from the centre of mass to the lower joint of link i.
θ_i: the angle between the vertical and link i.
I_i: moment of inertia of link i, with respect to the axis that passes through the COM of link i and perpendicular to the sagittal plane.
(x_e,y_e): the coordination of the swing leg tip.
(x_b,y_b): the coordination of the point of support leg.

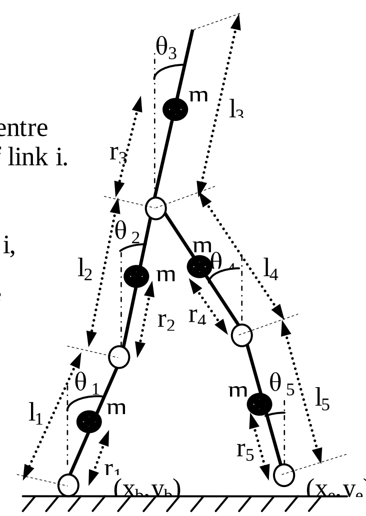

Figure 1

According to the kinematic relationship between the links of the robot that shown in figure (1), the position and velocity of the free end of the swing leg can be defined as follows:

$$x_e = \sum_{i=1}^{2}(l_i \sin\theta_i) + \sum_{i=4}^{5}(l_i \sin\theta_i) + x_b$$
$$y_e = \sum_{i=1}^{2}(l_i \cos\theta_i) + \sum_{i=4}^{5}(l_i \cos\theta_i) + y_b$$
$$\dot{x}_e = \sum_{i=1}^{2}(l_i\,\dot\theta_i \cos\theta_i) + \sum_{i=4}^{5}(l_i\dot\theta_i \cos\theta_i) + \dot{x}_b$$
$$\dot{y}_e = -\sum_{i=1}^{2}(l_i\,\dot\theta_i \sin\theta_i) + \sum_{i=4}^{5}(l_i\dot\theta_i \sin\theta_i) + \dot{y}_b$$

While the position and velocity of the centre of mass of each link is shown in the following form:

$$x_{ci} = \sum_{j=1}^{i-1}(bl_j \sin\theta_j) + \sum_{j=4}^{i}(l_j \sin\theta_j) + ar_i \sin\theta_i + x_b$$
$$y_{ci} = \sum_{j=1}^{i-1}(bl_j \cos\theta_j) + a\sum_{j=4}^{i}(l_j \cos\theta_j) + r_i \cos\theta_i + y_b$$
$$\dot{x}_{ci} = \sum_{j=1}^{i-1}(bl_j\dot\theta_j \cos\theta_j) + \sum_{j=4}^{i}(l_j\dot\theta_j \cos\theta_j) + ar_i\dot\theta_i \cos\theta_i + \dot{x}_b$$
$$\dot{y}_{ci} = \sum_{j=1}^{i-1}(bl_j\,\dot\theta_j \sin\theta_j) + a\sum_{j=4}^{i}(l_j\dot\theta_j \sin\theta_j) + r_i\dot\theta_i \sin\theta_i + \dot{y}_b$$

$$a = \begin{cases} 1 & ; i < 4 \\ -1 & ; i \geq 4 \end{cases} \qquad ; \qquad b = \begin{cases} 1 & ; j < 3 \\ 0 & ; j \geq 3 \end{cases}$$

The dynamic equation of the five-link biped robot is derived using Lagrange equations, where:

$$L = K - P$$

Where K is the kinetic energy, P is the potential energy and L is Lagrange coefficient.
The potential energy (P) is given by:

$$P = \sum_{i=1}^{5} P_i \qquad \text{with} \qquad P_i = m_i g y_{ci}$$

Where $g = 9.8\ m/s^2$ is the gravitational acceleration.
The kinetic energy is given by:

$$k = \sum_{i=1}^{5} K_i \qquad \text{with} \qquad K_i = \frac{1}{2} m_i\, v_{ci}{}^2 + \frac{1}{2}\, I_i\theta_i{}^2$$

Now the potential and kinetic energy will be substituted in Lagrange formula to solve the dynamic equations for the SSP:

$$\frac{d}{dt}\left\{\frac{\partial L}{\partial \dot{q}_i}\right\} - \frac{\partial L}{\partial q_i} = T_i$$

This can be expressed in the following form:

$$\frac{d}{dt}\left\{\frac{\partial K}{\partial \dot{q}_i}\right\} - \frac{\partial K}{\partial q_i} + \frac{\partial P}{\partial q_i} = T_i$$

Rearranging the equation above, the standard form of the equation of motion can be writing in the following form:

$$D(\theta)\ddot\theta + H(\theta,\dot\theta)\,\dot\theta + G(\theta) = T_\theta$$

Where $D(\theta)$ is 5*5 inertia matrix, $H(\theta)$ 5*5 centrifugal and coriolis' terms matrix, $G(\theta)$ is a 5*1 gravity matrix and $T_\theta, \theta, \dot\theta, \ddot\theta$ are 5*1 vectors of torque, generalized coordinates, velocities and accelerations respectively.

To control this dynamic model we should get a reference values to follow during the walking process. To get these values we have to drive the trajectories of the biped robot.

III. TRAJECTORIES PLANNING

Designing reference trajectories for the joints of the biped robot is one of the crucial aspects of motion control for these robots. Arbitrary planning of these trajectories may lead to instability due to tipping over during walking gait; it may also cause high energy consumption of the tracking actuators, that's why driving these trajectories became a very sensitive and important issue.

Mu and Wu (2004) have introduced a method for synthesizing the gait of a planar five-link biped robot walking on level ground for both the single support phase (SSP) and double support phase (DSP). They use time polynomial to produce limb and hip trajectories, which have the advantage of simplifying the problem by dividing the biped into three subsystems. They determine the joint angle profiles for a full gait cycle including the SSP and the DSP. The constraint functions and gait parameters are chosen to generate a repeatable gait.

First to derive the ankle trajectory a third and fourth order time polynomial have been chosen to represent the x_a and y_a coordinate of the ankle respectively.

$$X_a = \begin{cases} x_a(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3, \\ y_a(t) = b_0 + b_1 t + b_2 t^2 + b_3 t^3 + b_4 t^4 + b_5 t^5 \end{cases} \qquad (1)$$

Where $0 \leq t \leq T_s$; T_s the time period for SSP.

To solve the above equation the following ten constraints equations are going to be used:

$$y_a(0) = 0\,, \qquad y_a(T_s) = 0$$
$$x_a(T_m) = S_m\,, \qquad y_a(T_m) = H_m\,, \qquad \dot{y}_a(T_m) = 0$$
$$x_a(0) = -\frac{S_L}{2}, \qquad x_a(T_S) = S_L/2\,, \qquad \dot{x}_a(0) = 0$$
$$\dot{y}_a(0) = 0\,, \qquad \dot{x}_a(T_S) = 0\,, \qquad \dot{y}_a(T_S) = 0$$

Where: S_L is step length, T_s step period for the SSP, H_m is the maximum clearance of the swing limb, its location is S_m and T_m is the time corresponding to the maximum clearance.

Secondly, the hip trajectories are characterized using the following equation:

$$X_{hs} = \begin{cases} x_{hs}(t) = c_0 + c_1 t + c_2 t^2 + c_3 t^3, \\ y_{hs}(t) = y_h(t) \end{cases} \qquad (2)$$

Assuming that the height of the hips is kept constant during the gait and with the following constraints, equation (2) can be solved.

$$x_{hs}(0) = -S_{S0} \ , \qquad \dot{x}_{hs}(0) = V_{h1} \ ,$$

$$\dot{x}_{hs}(T_S) = V_{h2} \ , \qquad x_{hs}(T_S) = -S_{S0} + S_L$$

Where S_{S0} is the position of the hip at the beginning of the SSP, S_L is the step length, and V_{h1} is is the hip velocity at the beginning of each step, V_{h2} is the hip velocity at the end of the SSP.

The real challenge in designing these trajectories is in choosing appropriate hip velocities during the gait which is mainly a try and error process.

Lastly, the joint angle profiles can be determined uniquely, with the hip and swing limb tip trajectories already designed and the biped kinematic model, and can be described by the following equations:

$$\begin{cases} \theta_1(t) = \sin^{-1}\left(\dfrac{A_1C_1 + B_1\sqrt{A_1^2 + B_1^2 - C_1^2}}{A_1^2 + B_1^2}\right) \\ \theta_2(t) = \theta_1(t) + \sin^{-1}\left(\dfrac{A_1\cos(\theta_1(t)) - B_1\sin(\theta_1(t))}{l_2}\right) \\ \theta_3(t) = 0 \\ \theta_4(t) = \sin^{-1}\left(\dfrac{A_4C_4 + B_4\sqrt{A_4^2 + B_4^2 - C_4^2}}{A_4^2 + B_4^2}\right) \\ \theta_5(t) = \theta_4(t) + \sin^{-1}\left(\dfrac{A_4\cos(\theta_4(t)) - B_4\sin(\theta_4(t))}{l_5}\right) \end{cases}$$

Where

$$A_1 = x_{hs}(t), \qquad B_1 = y_{hs}(t), \qquad C_1 = \frac{A_1^2 + B_1^2 + l_1^2 - l_2^2}{2l_1},$$

$$A_4 = x_{as}(t) - x_{hs}(t), \qquad B_4 = y_{hs}(t) - y_{as}(t),$$

$$C_4 = \frac{A_4^2 + B_4^2 + l_4^2 - l_5^2}{2l_4}$$

IV. A NEW RECURRENT NEURAL NETWORK

Artificial Neural Network (ANN) provides effective techniques for system identification and control of nonlinear systems. As ANN have gained fame in solving problems with high difficulty as it have the ability to approximate nonlinear mappings and model complex system behavior; without prior knowledge of the system structure or parameters; to achieve accurate control through training.

Mainly there are two types of neural networks: feed-forward and recurrent neural network. Feed-forward neural network have no feedback elements, which mean that the output is calculated directly from the input. On the other hand, the recurrent neural networks have feedback connections which makes the output depends not only on the current input to the network, but also on the current (or previous) outputs or states of the network. Because of this, recurrent neural networks are considered more powerful than feed-forward networks, and have important uses in control applications.

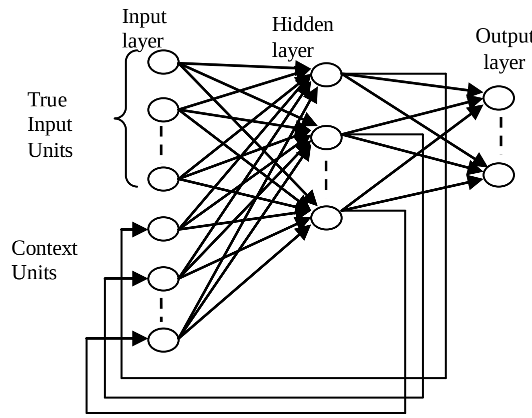

Figure 2: Elman Neural Network

Fig. 2 shows the structure of Elman neural network which is one of the early networks in this field. The main problem with it is that its training and speed of convergence is usually very slow.

In order to enhance the performance, a new recurrent neural network is introduced, which feedback the output of the network to both the hidden and output layer. The output of the neural network is used as a feedback signal due to its importance as the value to be adjusted to reach the desired value according to the specified input.

To analyze our network, a simplified model of the network which consists of one hidden layer and output layer, with zero activation value for the hidden layer and output layer is shown in Fig. 3. The weights of the feed-forward connections may vary, while the weights of the feed-back connections are fixed to reflect the previous situation of the network. The weights of the forward and backward paths are as shown in Fig. 3.

The equations which describe the relation between the input and output of the network are shown below:

$$y(k) = w_2x(k) + w_4r_1(k) + w_6r_2(k) \tag{3}$$

$$x(k) = w_1u(k) + w_3r_1(k) + w_5r_2(k) \tag{4}$$

$$r_1(k) = y(k-1) + \alpha r_1(k-1) \tag{5}$$

$$r_2(k) = y(k-1) + \beta r_2(k-1) \tag{6}$$

Using Z transforms on equations (1) to (4) above, we get:

$$y(z) = w_2x(z) + w_4r_1(z) + w_6r_2(z) \tag{7}$$

$$x(z) = w_1u(z) + w_3r_1(z) + w_5r_2(z) \tag{8}$$

$$Z\,r_1(k) = y(z) + \alpha r_1(z) \tag{9}$$

$$Z\,r_2(k) = y(z) + \beta r_2(z) \tag{10}$$

Equations (5) to (8) give the following transfer function:

$$y(z)\left[Z^2 - (\alpha + \beta + w_2\,w_3 + w_2\,w_5 + w_4 + w_6)Z + [\alpha\beta + (w_2\,w_3 + w_4)\beta + (w_2\,w_5 + w_6)\alpha]\right] = u(z)[w_1\,w_2\,Z^2 - w_1\,w_2(\alpha + \beta)Z + \alpha\,\beta] \tag{11}$$

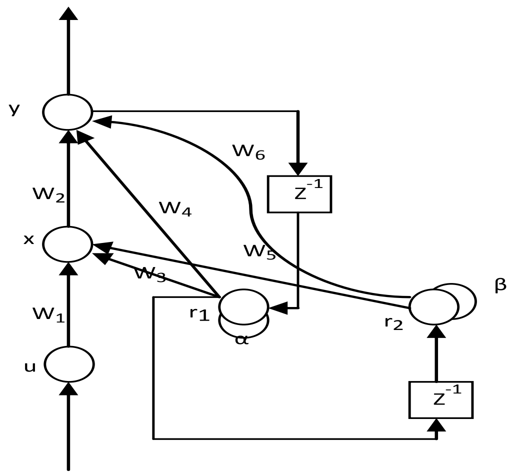

Figure (3)

Equation (11) above implies that:

$$y(k) = (\alpha + \beta + w_2\, w_3 + w_2\, w_5 + w_4 + w_6)y(k-1) - [\alpha\beta + (w_2\, w_3 + w_4)\beta + (w_2\, w_5 + w_6)\alpha]y(k-2) + w_1\, w_2\, u(k) - w_1\, w_2(\alpha+\beta)u(k-1) + \alpha\,\beta u(k-2) \quad (12)$$

If we compare equation (10) with the discrete form of the PID controller in equation (11); we can see that they have similar form; that implies that if we equate the terms of both equations, which shows our new RNN is similar in behavior to a PID controller.

$$y(k) = \frac{2T+\Delta T}{T+\Delta T} y(k-1) - \frac{T}{T+\Delta T} y(k-2) + \frac{(K_p+K_iT)\Delta T + K_D + K_pT + K_i\Delta T^2}{T+\Delta T} u(k) - \frac{(K_p+K_iT)\Delta T + 2K_D + 2K_pT + K_i\Delta T^2}{T+\Delta T} u(k-1) + \frac{K_D+K_pT}{T+\Delta T} u(k-2) \quad (11)$$

Where:

$$(\alpha + \beta + w_2\, w_3 + w_2\, w_5 + w_4 + w_6) = \frac{2T+\Delta T}{T+\Delta T}$$

$$[\alpha\beta + (w_2\, w_3 + w_4)\beta + (w_2\, w_5 + w_6)\alpha] = \frac{T}{T+\Delta T}$$

$$w_1\, w_2 = \frac{(K_p + K_iT)\Delta T + K_D + K_pT + K_i\Delta T^2}{T+\Delta T}$$

$$w_1\, w_2(\alpha+\beta) = \frac{(K_p + K_iT)\Delta T + 2K_D + 2K_pT + K_i\Delta T^2}{T+\Delta T}$$

$$\alpha\,\beta = \frac{K_D + K_pT}{T+\Delta T}$$

Where T the inertia time and ΔT is the sampling time.

V. Conclusion

From the above we can see that our new recurrent neural network give the same behavior as PID controller in addition the $\alpha\ and\ \beta$ factors can be used to increase the derivative and proportional parts in the PID controller while w_1 and w_2 can be used to reduce the integral part to give nearly a PD controller behavior.

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